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The Rev. Charles Graves communicated the following elementary geometrical proof of Joachimsthal's theorem.

LEMMA 1.—*If tangent planes be drawn at two points, P , P' , on a central surface of the second order ; and if perpendiculars be let fall from the points of contact on these tangent planes ; the perpendiculars will be proportional to the perpendiculars let fall from the centre of the surface upon the tangent planes.*

This is evident in the case of the sphere; and the theorem may be extended to the other surfaces by a simple *deformation*. Or it may be proved analytically in the simplest way, by means of the ordinary equation of the tangent plane.

LEMMA 2.—*Let LL' be the line of intersection of the two tangent planes, and let the point S be taken on it so that the lines PS , $P'S$, make equal angles with the line LL' ; then the lines PS , $P'S$, will be reciprocally proportional to the perpendiculars let fall from the centre upon the tangent planes at P and P' .*

For the lines PS , $P'S$, are evidently proportional to the perpendiculars let fall from P , P' , upon the tangent planes; and these, by the preceding Lemma, are proportional to the perpendiculars let fall from the centre upon the tangent planes at P' and P .

If the point S has been taken in L , L' , so that the angles PSL , $P'SL'$, are equal, the point S will be that the sum of whose distances from P and P' is a minimum.

Again, the lines PS , $P'S$, being tangents, are proportional to the parallel semi-diameters of the surface. We may, therefore, state the result at which we have now arrived in the following proposition.

If two points on a central surface be connected by a shortest line passing over the line of intersection of the two planes which touch the surface at those two points; the semi-diameters of the surface parallel to the two straight portions of the shortest line will be reciprocally proportional to the perpendiculars let fall from the centre upon the tangent planes in which those portions are respectively contained.